COMBINATORIA

Note Title 17/01/2012

A antipasti, P primi, S secondi

3 5 6

$$A \cdot P \cdot S = 90$$

CONTARE I DIVISORI >0 DI 144

$$144 = 2^4 \cdot 3^2$$

$$2^p \, 3^q \qquad \underbrace{0 \leq p \leq 4}_{5} \qquad \underbrace{0 \leq q \leq 2}_{3}$$

1 Oro = 13 monete d'argento

1 Ar = 17 bronzo

Prezzi < 2 ori?

$2 \cdot 13 \cdot 17 - 1$

O	A	B
		$\bullet$

$0 \leq \bullet \leq 16$

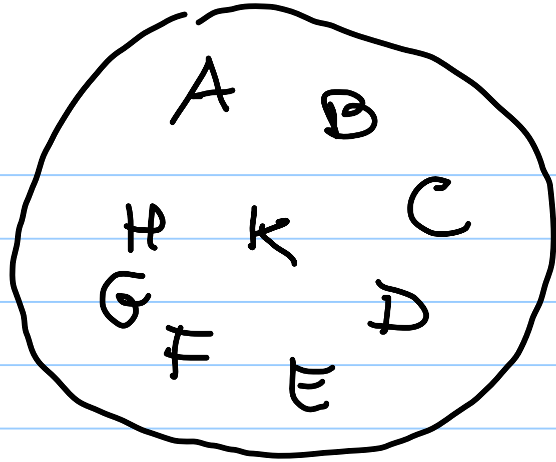
$0 \leq \bullet \leq 12$

$A = \{1, 2, 3\}$

6 modi

8 modi $= 2^3$

1	2	3
SI	NO	SI



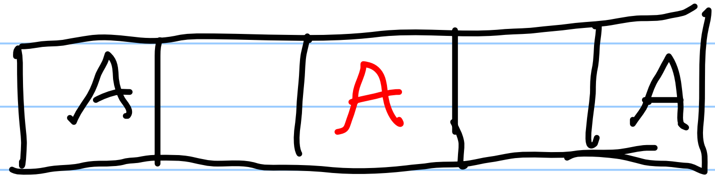
2^9

40 persone 40 sedie

$40! = 40 \cdot 39 \cdot 38 \cdot 37 \cdot \ldots \cdot 2 \cdot 1$

40^2

Aldo Berto



100 p. 40 sedie

$$\boxed{\binom{100}{40}} \cdot 40!$$

$$\parallel$$

$$\frac{100!}{60!} = 100 \cdot 99 \cdot \ldots \cdot 61 \cdot \frac{60!}{60!}$$

$\binom{n}{k}$ = il numero di modi in cui si possono scegliere k persone tra n

$$\frac{100!}{60! \cdot 40!} \qquad \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

$$\binom{n}{k} = \binom{n}{n-k}$$

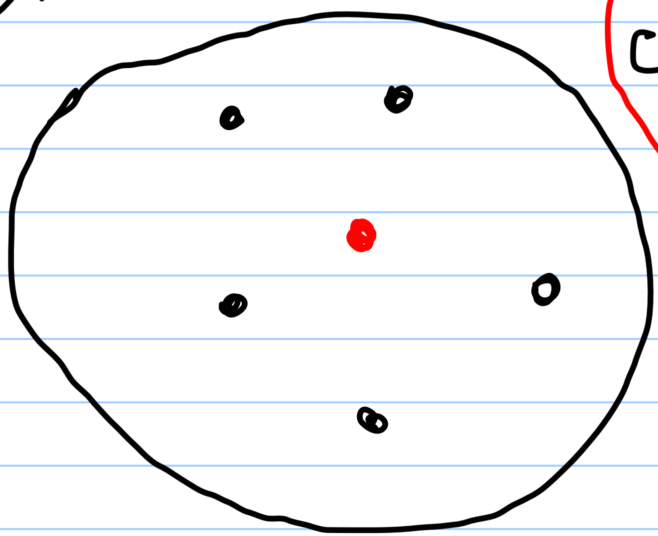
k fortunati si siedono

n-k Sfigati

$$\binom{n}{0} = 1$$

$$\binom{n+1}{k+1} = \binom{n}{k+1} + \binom{n}{k}$$

Prof. di Latino



classe $n+1$ persone

interrogati: $k+1$

Rosso si salva

Rosso prende 3

Double counting!

$\binom{5}{3} = 10$

$\binom{6}{3} = \underline{\underline{20}}$

$$\begin{array}{ccccccccccccccc}
 & & & & & & & \cancel{1} & & & & & & & 1 \\
 & & & & & & 0 \quad 1 & & 1 & & & & & & 2 \\
 & & & & & 0 \quad 1 & & 2 & & 1 & & & & & 4 \\
 & & & & 0 \quad 1 & & 3 & & 3 & & 1 & & & & 8 \\
 & & & 0 \quad 1 & & 4 & & 6 & & 4 & & 1 & & & 16 \\
 & & & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & 32 \\
 & & 1 & & 6 & & 15 & & 20 & & 15 & & 6 & \quad 1 & 64 \\
 & & & & \vdots & & & & & & \vdots & & & & \vdots
\end{array}$$

$$(x-\alpha)(x-\beta)\cdots(x-\zeta)$$

$$\binom{1}{0} + \binom{1}{1} = \binom{2}{1}$$

$$\binom{0}{0}$$

$$
\begin{array}{ccccccc}
 & & & 1 & & & \\
 & & 1 & & 1 & & \\
 & 1 & & 2 & & 1 & \\
1 & & 3 & & & & 1
\end{array}
$$

$$\binom{2}{0} + \binom{2}{1} = \binom{3}{1}$$

Sottoinsiemi $= 2^n$

$$\| $$

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n}$$

BINOMIO DI NEWTON

$$(a+b)^2 = 1a^2 + 2ab + 1b^2$$

$$(a+b)^3 = 1a^3 + 3a^2b + 3ab^2 + 1b^3$$

$$(a+b)^4 = a^4 + 4(\quad) + 6(\quad) + 4ab^3 + b^4$$

$$(a+b)(a+b)(a+b)(a+b) = \dots + \binom{4}{2} a^2 b^2 + \dots$$

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \dots$$

$$+ \binom{n}{n} b^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

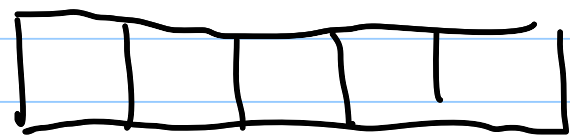
$$0 = (1-1)^n = \binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \dots$$

$$\pm 1$$

$$2^n = (1+1)^n = \binom{n}{0} + \binom{n}{1} + \dots + \binom{n}{n}$$

ANAGRAMMI

ABCDE



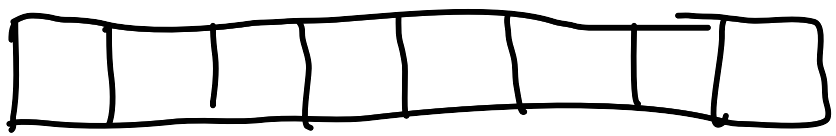
$\cancel{2^5}$ $5!$

MAMMA

$$\frac{5!}{3!\,2!} = \binom{5}{2}$$

AABBBCCC

$$\frac{8!}{2!\,3!\,3!}$$



$$\binom{8}{2} \cdot \binom{6}{3} \cdot \binom{3}{3}$$

$$\frac{8!}{2!\,\cancel{6!}} \times \frac{\cancel{6!}}{3!\,\cancel{3!}} \times \frac{\cancel{3!}}{3!}$$

$$\frac{8!}{2!\,3!\,3!}$$

A A B B B C C

AABCCC

- B $\longrightarrow \binom{5}{2}$
- C $\longrightarrow \dfrac{5!}{1!\,2!\,2!}$ BAACC
- A $\rightsquigarrow \dfrac{5!}{1!\,1!\,3!}$

$$\binom{5}{3} = \frac{5!}{3!\,2!}$$

$$\binom{5}{3,1,1} = \frac{5!}{3!\,1!\,1!}$$

$$(a+b+c)^4 = \dots + \binom{4}{2,1,1} a^2 b c + \dots$$

$$\sum_{i+j+k=n} \binom{n}{i,j,k} a^i b^j c^k$$

25 persone

8 Amst

7 Barcell

10 CHICAGO

$$\binom{25}{8,\ 7,\ 10}$$

1	2	...	23	24	25
		...			

8 SQUADRE (in CASA/FUORI)

$$\frac{8!}{4!} = \binom{8}{4} \cdot 4!$$

A
B
C
D

$3 \quad 1 \mid 2 \quad 5 \mid 7 \quad 6 \mid 4 \quad 8$

$$\frac{8!}{4!}$$

$$7 \cdot 5 \cdot 3 \cdot 1 \cdot 2^4$$

$\uparrow 8 \cdot 2$

ANAGRAMMI DI "MATEMATICA"

senza 3 "A" CONSECUTIVE

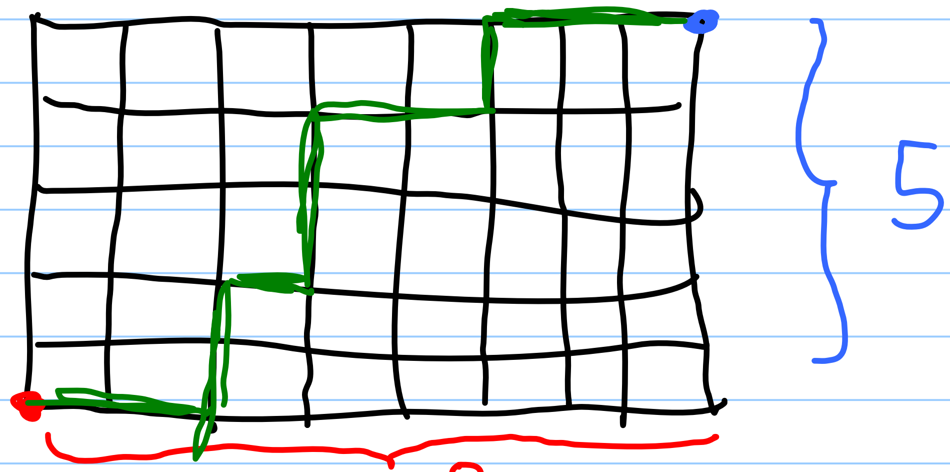
$$\underbrace{\binom{10}{3,2,2,1,1}}_{\text{tutti an.}} - \underbrace{\binom{8}{2,2,1,\dots,1}}_{\text{tre A consecutive}}$$

$X = AAA \qquad X\,M\,T\,E\,M\,T\,I\,C$

$$1\binom{n}{1} + 2\cdot\binom{n}{2} + \underbrace{3\binom{n}{3}}_{\substack{\text{scelgo 3 persone} \\ \text{e 1 dico che} \\ \text{e' il capo}}} + \dots + \binom{n}{n}\cdot n = n\cdot 2^{n-1}$$

(facile) Quante diagonali ha un poligono di n lati?

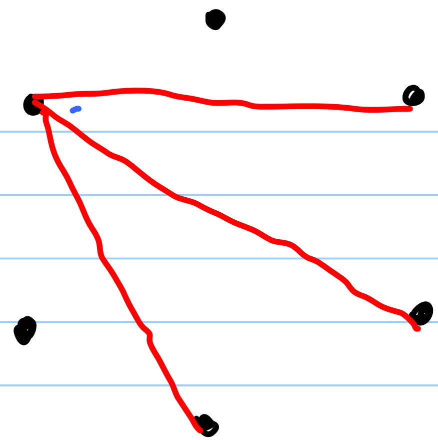
(medio)



Contare il numero dei percorsi di questo tipo, che vanno da Rosso a Blu muovendosi lungo la griglia e sempre verso l'alto o verso destra

(difficile) In quanti modi si può scrivere 30 come somma di 7 interi positivi? (Nota bene: l'ordine CONTA, cioè 1+5+1+1+1+1+20 è diverso da 5+1+1+1+1+1+20)

(medio) Trova il numero di anagrammi di COMPASSO in cui tra le due O ci sia esattamente una lettera



$$\frac{n(n-3)}{2}$$

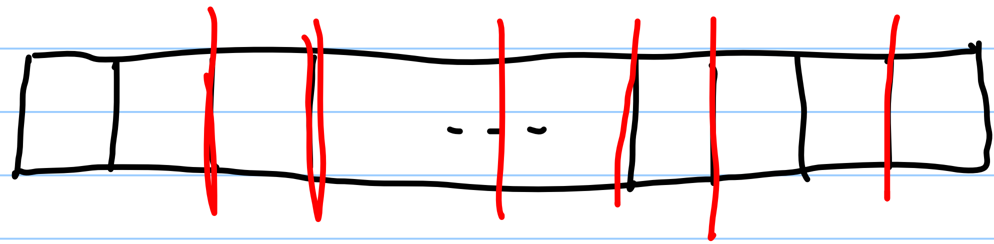
$$\binom{n}{2} - n = \frac{n!}{2!\,(n-2)!} - n$$

DIAG+LATI

$$= \frac{n(n-1)}{2} - n$$

$$\underbrace{DDSSDS\ldots}_{13} \qquad \binom{13}{5}$$

$30 =$ somma cose > 0



$$\binom{29}{6}$$

$1 + 0 + 3$

$0 + 0 + 4$

$$\blacksquare = \blacksquare$$

$$\binom{30+7-1}{7-1}$$

COMPASSO

$\underbrace{O \cdot O}_{X}$

$\cdot = S$

CMPASX

$5!$

$+$

$\cdot \neq S$

MPASS X

$\frac{6!}{2} \cdot 4$

$= 3 \cdot 6!$

$\boxed{O}\ \boxed{}\ \boxed{O}$

$6 \cdot \frac{6!}{2} = 3 \cdot 6!$